## Abstract

Spin glass and spin liquid states represent two distinct phases of disordered quantum systems, differing in whether spins freeze or exhibit quantum fluctuations. Despite sharing the common feature of disorder, these states have developed along entirely separate lines of study. For spin glasses, models with all-to-all random interactions, such as the Sherrington-Kirkpatrick (SK) model, have been intensively studied to clarify the fundamental nature of spin freezing. In contrast, spin liquid research has focused on models like Anderson's resonating valence bond and the Kitaev model, which emphasize the role of geometrically constrained and competing interactions. However, the emergence of the Sachdev-Ye-Kitaev (SYK) model in recent years has brought a new trend in the two research fields. The SYK model has all-to-all random interactions similar to the SK model, but the interactions are not two-body but four-body ones. Notably, due to the higher-order interactions, the SYK model does not exhibit spin glassy behavior but a chaotic state similar to spin liquid states. Given that a random quantum-spin Heisenberg model was also found to exhibit the SYK-like spin liquid in the intermediate temperature range, quantum spin models with higher-order interactions, inspired by the SYK model, are expected to provide insights that distinguish the behavior of spin glasses and spin liquids.

In this study, we aim to clarify the relation between the spin glass and spin liquid states and how they change with each other. For this purpose, we consider a model that feature all-to-all random interactions for p spins. The model includes several cases studied previously, especially connecting two cases: one with Ising-like one-component interactions and the other with isotropic three-component interactions. To clarify the conditions under which a spin liquid or glass state emerges in the ground state, we systematically analyze the system size N dependence of the Edwards-Anderson (EA) order parameter using exact diagonalization. As the system size grows, we discover a crossover in which the EA order parameter changes from a spin-liquid-like decrease to a spin-glass-like increase. By using the point where the EA order parameter reaches its minimum, we construct a phase diagram, and reveal consistent behavior across multiple values of p. Furthermore, we analytically generalize the parameters that determine the shape of the density of states in the isotropic case to an arbitrary number of components. By comparing

the crossover of the density of states with the phase boundaries obtained from the EA order parameter, an excellent agreement between them is found. In addition, to explore the connection to the ground state properties, the spectral statistics are examined. We find that changes in the level spacing ratio distributions do not correspond to the emergence of spin liquid or glass behavior. Instead, these changes merely reproduce the distinction between whether the model is one-component or not.