Abstract

Just as a collection of elementary particles forms an atom, a collection of atoms forms a material, and collections of materials form life, stars, and galaxies, hierarchical structures are a universal characteristic found throughout nature, with each level showing its unique and striking functionality. Research into such hierarchies is extremely important in material science for developing new functional materials and devices leveraging emergent physical properties. Recently, this research has gained a significant interest, particularly at the intersection of magnetism and topology. For instance, magnetic skyrmions and skyrmion strings can assemble into superstructures known as skyrmion crystals through effective interactions among skyrmions. They exhibit intriguing collective spin dynamics and unconventional electromagnetic phenomena, leading to numerous proposals for device applications utilizing them. Furthermore, magnetic hopfions, one of the topological spin knot structures, can also be regarded as superstructures constructed from closed loops of twisted skyrmion strings. Their remarkable stability arising from the linking topology of the knot theory, and transport phenomena and three-dimensional dynamics unattainable in simple skyrmions and their crystals, have been increasingly unveiled, solidifying their presence as a new member of the topological magnetic hierarchy. Given the unique and fascinating functionality of each topological structure in such hierarchy, it is natural to further explore the stability of superstructures formed by multiple hopfions through their interactions and what kind of nontrivial spin dynamics and quantum transport the hopfion systems would manifest. Since a simple way to excite hopfions using electric current has recently been reported, understanding of the competing phenomena of interacting hopfions will undoubtedly be the next important research topic for further advancement of the topological magnetism.

In this thesis, we present three key findings that significantly advance our understanding of interacting magnetic hopfions and their static and dynamic properties. First, we examine the fundamental interactions between hopfions for various relative configurations. Considering an isotropic

frustrated spin model, we elucidate that the effective interactions between horizontally arranged hopfions depend on the relative angle between them, determining whether the interaction is long-range repulsive or attractive, which can be summarized as a "color rule". In the case of the attraction, the hopfions eventually cause a fusion into those with higher Hopf number or pair annihilate to a topologically trivial magnetic state. We also analyze the interactions of vertically aligned hopfions and find that they stabilize a hopfion chain, a periodic one-dimensional structure of hopfions.

Next, leveraging these knowledge, we construct the first example of a stable superstructure where the hopfions are arranged periodically in three-dimensional space. We discover that additional in-plane magnetic anisotropy plays a crucial role in stabilizing the crystalline superstructure. The superstructures is composed of hopfion chains running perpendicularly to the anisotropic plane, each threaded by a string of half-skyrmions known as magnetic merons. It exhibits unique magnetic and topological properties in both real and momentum space, which could be identified through neutron scattering experiments and quantum transport measurements.

Finally, we unveil non-equilibrium dynamics of hopfions driven by the spin-orbit torque (SOT). For a hopfion with $H = 1$, we show that the SOT causes translational and rotational motions of the hopfion, depending on its helicity. For hopfions with higher Hopf numbers, we find that the SOT works as an effective tension on them, causing separation into multiple hopfions with lower Hopf numbers. From systematic studies while changing the magnetic field and the strength of the SOT, we elucidate a hierarchical structure in the non-equilibrium steady-state phase diagrams between the hopfions with different Hopf numbers. These results demonstrate that the SOT not only generates the motion of hopfions but also enables the control of their topological properties.

Thus, through this thesis, we clarify the effective interactions between magnetic hopfions, the stability of hopfion superstructures, and the non-equilibrium dynamics of hopfions driven by the SOT. These are pioneering results that demonstrate the existence of abundant phenomena not only in a single hopfion but also in multiple hopfions interacting with each other. We believe that our fingings open the next paradigm in topological magnetism and stimulate further theoretical and experimental studies on hopfions, leading to unprecedented emergent phenomena and collective spin dynamics applicable for the next-generation electromagnetic devices.